丘维声《高等代数》上册习题整理

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1 线性方程组

1.1 解线性方程组的矩阵消元法

习题1.1(1) 解线性方程组

$$\begin{cases} x_1 - 3x_2 - 2x_3 = 3, \\ -2x_1 + x_2 - 4x_3 = -9, \\ -x_1 + 4x_2 - x_3 = -7. \end{cases}$$

解答：

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 3 \\ -2 & 1 & -4 & -9 \\ -1 & 4 & -1 & -7 \end{array}\right] \xrightarrow[R_3 \leftarrow R_3 + R_1]{R_2 \leftarrow R_2 + 2R_1} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 3 \\ 0 & -5 & -8 & -3 \\ 0 & 1 & -3 & -4 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 3 \\ 0 & 1 & -3 & -4 \\ 0 & -5 & -8 & -3 \end{array}\right]$$

$$\xrightarrow{R_3 \leftarrow R_3 + 5R_2} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 3 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & -23 & -23 \end{array}\right] \xrightarrow{R_3 \leftarrow -\frac{1}{23} R_3} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 3 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow[R_1 \leftarrow R_1 + 2R_3]{R_2 \leftarrow R_2 + 3R_3} \left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \leftarrow R_1 + 3R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right].$$

故

$$(x_1, x_2, x_3)^T = (2, -1, 1)^T. \ \square$$

习题1.1(2) 解线性方程组

$$\begin{cases} x_1 + 3x_2 + 2x_3 = 1, \\ 2x_1 + 5x_2 + 5x_3 = 7, \\ 3x_1 + 7x_2 + x_3 = -8, \\ -x_1 - 4x_2 + x_3 = 10. \end{cases}$$

解答：

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 2 & 5 & 5 & 7 \\ 3 & 7 & 1 & -8 \\ -1 & -4 & 1 & 10 \end{array}\right] \xrightarrow[\substack{R_3 \leftarrow R_3 - 3R_1 \\ R_4 \leftarrow R_4 + R_1}]{R_2 \leftarrow R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & -1 & 1 & 5 \\ 0 & -2 & -5 & -11 \\ 0 & -1 & 3 & 11 \end{array}\right] \xrightarrow{R_2 \leftarrow -R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & -1 & -5 \\ 0 & -2 & -5 & -11 \\ 0 & -1 & 3 & 11 \end{array}\right]$$

$$\xrightarrow[R_4 \leftarrow R_4 + R_2]{R_3 \leftarrow R_3 + 2R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & -7 & -21 \\ 0 & 0 & 2 & 6 \end{array}\right] \xrightarrow{R_3 \leftarrow -\frac{1}{7}R_3} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 6 \end{array}\right] \xrightarrow[R_2 \leftarrow R_2 + R_3]{R_4 \leftarrow R_4 - 2R_3} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow[R_1 \leftarrow R_1 - 3R_2]{R_1 \leftarrow R_1 - 2R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

故

$$(x_1, x_2, x_3)^T = (1, -2, 3)^T. \ \square$$

习题1.1(3)　解线性方程组

$$\begin{cases} x_1 - 3x_2 - 2x_3 - x_4 = 6, \\ 3x_1 - 8x_2 + x_3 + 5x_4 = 0, \\ -2x_1 + x_2 - 4x_3 + x_4 = -12, \\ -x_1 + 4x_2 - x_3 - 3x_4 = 2. \end{cases}$$

解答:

$$\left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 3 & -8 & 1 & 5 & 0 \\ -2 & 1 & -4 & 1 & -12 \\ -1 & 4 & -1 & -3 & 2 \end{array}\right] \xrightarrow[\substack{R_3 \leftarrow R_3 + 2R_1 \\ R_4 \leftarrow R_4 + R_1}]{R_2 \leftarrow R_2 - 3R_1} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & -5 & -8 & -1 & 0 \\ 0 & 1 & -3 & -4 & 8 \end{array}\right]$$

$$\xrightarrow[R_4 \leftarrow R_4 - R_2]{R_3 \leftarrow R_3 + 5R_2} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & 27 & 39 & -90 \\ 0 & 0 & -10 & -12 & 26 \end{array}\right] \xrightarrow{R_3 \leftrightarrow R_4} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & -10 & -12 & 26 \\ 0 & 0 & 27 & 39 & -90 \end{array}\right]$$

$$\xrightarrow{R_3 \leftarrow -\frac{1}{2}R_3} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & 5 & 6 & -13 \\ 0 & 0 & 27 & 39 & -90 \end{array}\right] \xrightarrow{R_4 \leftarrow R_4 - \frac{27}{5}R_3} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & 5 & 6 & -13 \\ 0 & 0 & 0 & \frac{33}{5} & -\frac{99}{5} \end{array}\right] \xrightarrow{R_4 \leftarrow \frac{5}{33}R_4} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & 5 & 6 & -13 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

$$\xrightarrow[R_3 \leftarrow \frac{1}{5}R_3]{R_3 \leftarrow R_3 - 6R_4} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right] \xrightarrow[R_2 \leftarrow R_2 - 7R_3]{R_2 \leftarrow R_2 - 8R_4} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right] \xrightarrow[\substack{R_1 \leftarrow R_1 + 2R_3 \\ R_1 \leftarrow R_1 + 3R_2}]{R_1 \leftarrow R_1 + R_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right].$$

故

$$(x_1, x_2, x_3, x_4)^T = (2, -1, 1, -3)^T. \ \square$$

习题1.1(4)　解线性方程组

$$\begin{cases} x_1+3x_2-7x_3=-8, \\ 2x_1+5x_2+4x_3=4, \\ -3x_1-7x_2-2x_3=-3, \\ x_1+4x_2-12x_3=-15. \end{cases}$$

解答:

$$\left[\begin{array}{cccc|c} 1 & 3 & -7 & -8 \\ 2 & 5 & 4 & 4 \\ -3 & -7 & -2 & -3 \\ 1 & 4 & -12 & -15 \end{array}\right] \xrightarrow[R_4\leftarrow R_4-R_1]{\substack{R_2\leftarrow R_2-2R_1 \\ R_3\leftarrow R_3+3R_1}} \left[\begin{array}{ccc|c} 1 & 3 & -7 & -8 \\ 0 & -1 & 18 & 20 \\ 0 & 2 & -23 & -27 \\ 0 & 1 & -5 & -7 \end{array}\right] \xrightarrow{R_2\leftrightarrow R_4} \left[\begin{array}{ccc|c} 1 & 3 & -7 & -8 \\ 0 & 1 & -5 & -7 \\ 0 & 2 & -23 & -27 \\ 0 & -1 & 18 & 20 \end{array}\right]$$

$$\xrightarrow[R_4\leftarrow R_4+R_2]{R_3\leftarrow R_3-2R_2} \left[\begin{array}{ccc|c} 1 & 3 & -7 & -8 \\ 0 & 1 & -5 & -7 \\ 0 & 0 & -13 & -13 \\ 0 & 0 & 13 & 13 \end{array}\right] \xrightarrow{R_3\leftarrow -\frac{1}{13}R_3} \left[\begin{array}{ccc|c} 1 & 3 & -7 & -8 \\ 0 & 1 & -5 & -7 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 13 & 13 \end{array}\right] \xrightarrow{R_4\leftarrow R_4-13R_3} \left[\begin{array}{ccc|c} 1 & 3 & -7 & -8 \\ 0 & 1 & -5 & -7 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow[R_1\leftarrow R_1-3R_2]{\substack{R_2\leftarrow R_2+5R_3 \\ R_1\leftarrow R_1+7R_3}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

故

$$(x_1,x_2,x_3)^T=(5,-2,1)^T. \ \square$$

习题1.1(5)　解线性方程组

$$\begin{cases} x_1-2x_2+3x_3-4x_4=4, \\ x_1+x_2-x_3+x_4=-11, \\ x_1+3x_2+x_4=1, \\ -7x_2+3x_3+x_4=-3. \end{cases}$$

解答:

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 1 & 1 & -1 & 1 & -11 \\ 1 & 3 & 0 & 1 & 1 \\ 0 & -7 & 3 & 1 & -3 \end{array}\right] \xrightarrow[R_3\leftarrow R_3-R_1]{R_2\leftarrow R_2-R_1} \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 3 & -4 & 5 & -15 \\ 0 & 5 & -3 & 5 & -3 \\ 0 & -7 & 3 & 1 & -3 \end{array}\right] \xrightarrow{R_2\leftarrow \frac{1}{3}R_2} \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -\frac{4}{3} & \frac{5}{3} & -5 \\ 0 & 5 & -3 & 5 & -3 \\ 0 & -7 & 3 & 1 & -3 \end{array}\right]$$

$$\xrightarrow[R_4\leftarrow R_4+7R_2]{R_3\leftarrow R_3-5R_2} \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -\frac{4}{3} & \frac{5}{3} & -5 \\ 0 & 0 & \frac{11}{3} & -\frac{10}{3} & 22 \\ 0 & 0 & -\frac{19}{3} & \frac{38}{3} & -38 \end{array}\right] \xrightarrow[R_4\leftarrow 3R_4]{R_3\leftarrow 3R_3} \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -\frac{4}{3} & \frac{5}{3} & -5 \\ 0 & 0 & 11 & -10 & 66 \\ 0 & 0 & -19 & 38 & -114 \end{array}\right]$$

$$
\xrightarrow{R_4 \leftarrow 11R_4 + 19R_3}
\left[\begin{array}{cccc|c}
1 & -2 & 3 & -4 & 4 \\
0 & 1 & -\frac{4}{3} & \frac{5}{3} & -5 \\
0 & 0 & 11 & -10 & 66 \\
0 & 0 & 0 & 228 & 0
\end{array}\right]
\xrightarrow{R_4 \leftarrow \frac{1}{228} R_4}
\left[\begin{array}{cccc|c}
1 & -2 & 3 & -4 & 4 \\
0 & 1 & -\frac{4}{3} & \frac{5}{3} & -5 \\
0 & 0 & 11 & -10 & 66 \\
0 & 0 & 0 & 1 & 0
\end{array}\right]
$$

$$
\xrightarrow[R_3 \leftarrow \frac{1}{11} R_3]{R_3 \leftarrow R_3 + 10R_4}
\left[\begin{array}{cccc|c}
1 & -2 & 3 & -4 & 4 \\
0 & 1 & -\frac{4}{3} & \frac{5}{3} & -5 \\
0 & 0 & 1 & 0 & 6 \\
0 & 0 & 0 & 1 & 0
\end{array}\right]
\xrightarrow[R_2 \leftarrow R_2 - \frac{5}{3} R_4]{R_2 \leftarrow R_2 + \frac{4}{3} R_3}
\left[\begin{array}{cccc|c}
1 & -2 & 3 & -4 & 4 \\
0 & 1 & 0 & 0 & 3 \\
0 & 0 & 1 & 0 & 6 \\
0 & 0 & 0 & 1 & 0
\end{array}\right]
\xrightarrow[R_1 \leftarrow R_1 + 4R_4]{\substack{R_1 \leftarrow R_1 + 2R_2 \\ R_1 \leftarrow R_1 - 3R_3}}
\left[\begin{array}{cccc|c}
1 & 0 & 0 & 0 & -8 \\
0 & 1 & 0 & 0 & 3 \\
0 & 0 & 1 & 0 & 6 \\
0 & 0 & 0 & 1 & 0
\end{array}\right].
$$

故

$$(x_1, x_2, x_3, x_4)^T = (-8, 3, 6, 0)^T. \ \square$$